This follows as a special case of Pascal's Theorem for a hexagon inscribed in a conic.* The conic is in this case degenerate, two straight lines. But (23), (56) are parallel and hence must have an infinitely distant point in common with PQ. Therefore PQ is parallel to AA'.

Also solved by MARJORIE L. BROWN, O. S. ADAMS (two methods), F. E. WOOD, NATHAN ALTSHILLER, HANNAH SUFFIN, and H. H. CONWELL.

CALCULUS.

416. Proposed by CHARLES N. SCHMALL, New York City.

If A be a point on a cycloid and C the corresponding position of the center of the generating circle, show that AC envelops another cycloid half the size of the first.

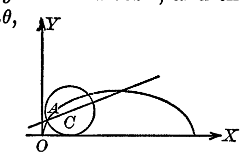
SOLUTION BY A. M. HARDING, University of Arkansas.

The coördinates of any point on the cycloid are $x = a\theta - a\sin\theta$, $y = a - a\cos\theta$, and the coördinates of the center of the generating circle are given by $x = a\theta$, $y = a$. The equation of AC is

$$\frac{x - a\theta}{\sin\theta} = \frac{y - a}{\cos\theta},$$

or

$$y - a = \cot\theta(x - a\theta).$$



The equation of a cycloid half the size of the given cycloid and having a cusp at O is

$$x = \frac{a\varphi}{2} - \frac{a}{2}\sin\varphi, \qquad y = \frac{a}{2} - \frac{a}{2}\cos\varphi.$$

We propose to show that AC is always tangent to this cycloid.

The equation of any tangent to this cycloid is

$$y - \frac{a}{2}(1 - \cos\varphi) = \cot\frac{\varphi}{2}\left[x - \frac{a}{2}(\varphi - \sin\varphi)\right].$$

Let $\varphi = 2\theta$. This equation then becomes

$$y - a\sin^2\theta = \cot\theta[x - a\theta + a\sin\theta\cos\theta]$$

or

$$y - a = \cot\theta(x - a\theta),$$

which is the same as the equation of AC.

Also solved by C. N. SCHMALL, ELIJAH SWIFT, G. W. HARTWELL, HORACE OLSON, O. S. ADAMS, M. R. GAFFET, R. H. HOWARD, J. B. REYNOLDS, and SHIMPEI NISHIMURA.

417. Proposed by H. S. UHLER, Yale University.

To the degree of approximation indicated show that $(\sqrt{-1})^{\sqrt{-1}} = 0.207879576351$.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

It is not difficult to show, as required in Todhunter's *Plane Trigonometry*, Ed. 1913, pp. 320–21, Examples 266, 275, "that $(a + bi)^{\alpha+\beta i}$ will be wholly real or imaginary if

$$(\beta/2)\log(a^2 + b^2) + \alpha\tan^{-1}(b/a)$$

is (I) zero, or an even multiple of $\pi/2$; or (II) an odd multiple $\pi/2$". In the problem, $a = \alpha = 0$, and $b = \beta = 1$, the conditions corresponding to (I), 0 being called an even number. $(\sqrt{-1})^{\sqrt{-1}} = e^{-\frac{1}{2}\pi}$, and the numerical value can be tested by using enough decimals in the values of e and π.

* Cremona, *Elements of Projective Geometry*, translated by Leudensdorf, Articles 88 and 153, 3d edition.